

Direct optical detection of pure spin current in semiconductors

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We suggest a new practical scheme for the direct detection of pure spin current by using the two-color Faraday rotation of optical quantum interference process (QUIP) in a semiconductor system. We demonstrate theoretically that the Faraday rotation of QUIP depends sensitively on the spin orientation and wave vector of the carriers, and can be tuned by the relative phase and the polarization direction of the ω and 2ω laser beams. By adjusting these parameters, the magnitude and direction of the spin current can be detected.

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Generating spin population in semiconductors is one of central goals of spintronics and has attracted a rapidly growing interest for its potential application in spintronic devices. For pure spin current, the spin-up and spin-down electron currents are expected to have equal magnitudes but travel in opposite directions ($\pm\mathbf{k}$). In semiconductors, these currents can be generated utilizing the spin Hall effect (SHE), [1, 2, 3, 4] optical quantum mechanical interference control between one- and two-photon excitations, [5, 6, 7, 8, 9] and one-photon absorption in the noncentrosymmetric semiconductors. [10, 11] However, the experimental measurement of spin current is an extremely challenging task. The pure spin current can only be detected indirectly in semiconductors by measuring the spin accumulation [4, 5, 6, 7, 12] near the boundary of the sample, the nonuniform spatial distribution, i.e., spin wavepacket [11]. So far, the direct measurement of a pure spin current, with *uniform* spatial spin and charge distributions, has still not been reported in semiconductor structures since the pure spin current exhibits vanishing charge current and total spin. The vanishing charge current and total spin make *direct* electrical or optical detection in semiconductor structures extremely difficult.

Is it possible to detect the *uniform* pure spin current in a semiconductor system? Although the total spin of a uniform pure spin current vanishes everywhere in the real space, the spin population is asymmetric in the momentum space, i.e., the asymmetric distribution of the spin-up (-down) electrons at opposite $\pm\mathbf{k}$ points. Direct optical detection would be possible if the optical transition at $\pm\mathbf{k}$ points become asymmetric utilizing a quantum interference process (QUIP), e.g., the two-photon process. The momentum-resolved optical transition can be detected by circularly polarized light, Faraday or Kerr rotation (FR or KR), and magnetic circular dichroism (MCD). In this work, we focus on the Faraday rotation spectrum because it is a powerful tool for investigating the spin dynamics of carriers in semiconductors [13]. The conventional FR vanishes for the detection of pure spin current since it is determined by the total spin of the system, i.e., the difference of the spin-up and spin-down

electron populations.

In this Letter, we suggest a new scheme utilizing a momentum-resolved two-color FR of optical quantum interference process for direct detection of spin current in a semiconductor structure. In the quantum interference process between one- and two-photon absorptions, the transition rate depends not only on the polarizations of the laser pulses, but also on the relative phase of the ω and 2ω excitations, which makes the transition rate depending on electron wave vector \mathbf{k} . The optical transition of QUIP can be enhanced at $+\mathbf{k}$ but vanishes at $-\mathbf{k}$, or *vice versa*. Therefore it is possible to transfer the angular momentum between the photon and the electron-hole pair at a specific \mathbf{k} point, while it is forbidden at the opposite $-\mathbf{k}$ point. Thus, the FR of QUIP can be used to detect the spin population at opposite $\pm\mathbf{k}$ points independently, i.e., the pure spin current case, by adjusting the external parameter such as the relative phase between the ω and 2ω excitations.

Supposing there is a pure spin current in semiconductor structures, we study the FR of a two-photon (ω and 2ω) quantum interference process. First, instead of perturbation theory, we calculate the transition rate for the one- and two-photon absorptions using the Volkov-type wave function which describes the electron state under an optical field of arbitrary intensity [14]. Then, we give the analytical expression of the FR of QUIP in a 3D sample. The valence and conduction states can be expressed as a

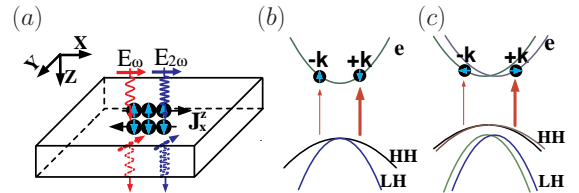


FIG. 1: (color online) Schematic illustration of two parallelly polarized beams propagating in semiconductor structures in the presence of a uniform pure spin current (a). The band structure of a 3D sample without the SOI (b) and 2D quantum well with the SOIs (c). The thickness of the red vertical arrows indicates the strength of the transitions of QUIP.

Volkov-type dressed state

$$\psi_{c,v}(\mathbf{k}, \mathbf{r}, t) = u_{c,v}(\mathbf{k}, \mathbf{r}) \exp[i\mathbf{k} \cdot \mathbf{r} - i\omega_{c,v}t] + \frac{ie}{m_{c,v}} \int_0^t \mathbf{k} \cdot \mathbf{A}(\tau) d\tau, \quad (1)$$

where c (v) refers to the conduction (valence) band, $u_{c,v}(\mathbf{k}, \mathbf{r})$ are the band-edge Bloch wave functions, $E_{c,v}(k) = \hbar\omega_{c,v}(k)$ the energy of the valence or conduction band, $m_{c,v}$ the effective masses, and \mathbf{A} is the total vector potential $\mathbf{A} = \mathbf{a}_1 A_1 \cos(\omega t + \varphi_\omega) + \mathbf{a}_2 A_2 \cos(2\omega t + \varphi_{2\omega})$. The excitation field satisfies the relation $\hbar\omega < E_g < \hbar 2\omega < E_g + \Delta_0$, so that the ω and 2ω excitations generate independent carriers through one- and two-photon absorption. The transition rate is calculated using a S-matrix formalism [14]

$$S = -\frac{i}{\hbar} \int_{-\infty}^{\infty} dt \int d^3r \psi_c^*(\mathbf{k}, \mathbf{r}, t) H_{int} \psi_v(\mathbf{k}, \mathbf{r}, t). \quad (2)$$

The transition rate is

$$W(\mathbf{k}) = C \left\{ \left(\frac{\eta_1}{2} \right)^2 |\mathbf{P}_{cv} \cdot \mathbf{a}_1|^2 A_1^2 + |\mathbf{P}_{cv} \cdot \mathbf{a}_2|^2 A_2^2 + \left[A_1 A_2 \frac{\eta_1}{2} (\mathbf{P}_{cv} \cdot \mathbf{a}_1)^* (\mathbf{P}_{cv} \cdot \mathbf{a}_2) e^{i(2\varphi_\omega - \varphi_{2\omega})} + c.c. \right] \right\}, \quad (3)$$

where $C = 2\pi \left(\frac{e}{2\hbar m_0 c} \right) \delta[\omega_{cv}(k) - 2\omega] (f_v - f_c)$, f_v (f_c) is the Fermi function of valence (conduction) electrons, $\mathbf{P}_{cv} = \langle c | \hat{p} | v \rangle$, $\eta_1 = \frac{eA_1}{\omega c m_{cv}} \mathbf{k} \cdot \mathbf{a}_1$, and $1/m_{cv} = 1/m_c - 1/m_v$. The term in the square brackets describes the quantum interference between the one- and two-photon excitations. From Eq. (3), the quantum interference term depends sensitively on the electron wave vector \mathbf{k} , the electron spin orientation, the polarization, and the relative phase of the pulses. The transition rate can be strongly asymmetric for the transitions at $\pm\mathbf{k}$, e.g., $W(-\mathbf{k}) = 0$ while $W(+\mathbf{k}) \neq 0$ by adjusting the polarization and the relative phase of the pulses.

The electron and hole states in Eq.(2) can be obtained from the single-band and the multiband Luttinger-Kohn (LK) effective-mass Hamiltonian, respectively. [15] The LK Hamiltonian reads

$$H_h(z_h, \rho) = \frac{\hbar^2 k^2}{2m_0} \begin{pmatrix} H_{hh} & L & M & 0 \\ L^* & H_{lh} & 0 & M \\ M^* & 0 & H_{lh} & -L \\ 0 & M^* & -L^* & H_{hh} \end{pmatrix}, \quad (4)$$

where:

$$H_{hh} = -\gamma_1 - \gamma_2 (\sin^2 \theta_e - 2 \cos^2 \theta_e),$$

$$H_{lh} = -\gamma_1 + \gamma_2 (\sin^2 \theta_e - 2 \cos^2 \theta_e),$$

$$M = -\sqrt{3} \gamma_2 \sin^2 \theta_e e^{-2i\varphi_e},$$

$$L = i2\sqrt{3} \gamma_3 \sin \theta_e \cos \theta_e e^{-i\varphi_e},$$

where $\mathbf{k} = (k, \theta_e, \varphi_e)$, γ_1 , γ_2 , and γ_3 are the Luttinger parameters. The energy dispersions of valence subbands in the isotropic approximation $\gamma_2 = \gamma_3$

are $E_{\pm} = \frac{\hbar^2 k^2}{2m_0} [-\gamma_1 \pm (R_h^2 + |L|^2 + |M|^2)^{1/2}]$, and $R_h = -\gamma_2 (\sin^2 \theta_e - 2 \cos^2 \theta_e)$. The eigenstates of the heavy-hole (hh) and light-hole (lh) bands are $|hh_+\rangle = \frac{1}{c_h} (R_h + E'_+, L^*, M^*, 0)^T$, $|hh_-\rangle = \frac{1}{c_h} (0, M, -L, R_h + E'_+)^T$, and $|lh_+\rangle = \frac{1}{c_h} (L, -R_h + E'_-, 0, M^*)^T$, $|lh_-\rangle = \frac{1}{c_h} (M, 0, -R_h + E'_-, -L^*)^T$, respectively, where c_h is normalization constant, and $E'_{\pm} = \pm \sqrt{|M|^2 + |L|^2 + R_h^2}$ is the energy difference between the hh and lh bands.

As shown schematically in Fig. 1(a), the ω and 2ω two-color optical fields are linearly parallel polarized along the x axis and propagate along $+z$, $\mathbf{E}(\mathbf{r}, t) = E_\omega e^{i(\omega t - \mathbf{k}_\omega \cdot \mathbf{r} + \varphi_\omega)} \hat{e}_\omega + E_{2\omega} e^{i(2\omega t - \mathbf{k}_{2\omega} \cdot \mathbf{r} + \varphi_{2\omega})} \hat{e}_{2\omega}$, and \hat{e}_ω and $\hat{e}_{2\omega}$ are the unit polarization vectors. The propagation directions, \mathbf{k}_ω and $\mathbf{k}_{2\omega}$ are both along $+\hat{z}$. Using Eq. (3), the difference of the refractive index for the right- and left- circular polarized lights in the presence of the pure spin current can be written as

$$N_+ - N_- \propto \sum_{\mathbf{k}} W_+(\mathbf{k}) + W_+(-\mathbf{k}) - W_-(\mathbf{k}) - W_-(-\mathbf{k}) \\ = \sum_{\mathbf{k}} C_0 [C_{i1} (f_u + f_d) \cos(2\varphi_\omega - \varphi_{2\omega}) + C_{i2} \sin(2\varphi_\omega - \varphi_{2\omega}) \\ \times \sin(2\varphi_e) (f_d - f_u) + C_l (f_u - f_d)] \times \delta[\omega_{cv}(k) - 2\omega], \quad (5)$$

where

$$C_0 = 2\pi \frac{2}{\sqrt{2} c_h^2} \left(\frac{e}{\hbar 2m_0 c} \right)^2 P^2,$$

$$C_{i1} = \frac{\eta'_1}{2} A_1 A_2 k \sin \theta_e \cos \varphi_e \Re_{h,l},$$

$$C_{i2} = \frac{\eta'_1}{2} A_1 A_2 k \sin \theta_e \cos \varphi_e \Im_{l,h} \Re(M) / \sqrt{3},$$

$$C_l = A_2^2 \Re_{h,l},$$

where $\Re_h = [(R_h + E'_+)^2 - (|M|^2 + |L|^2)/3]$ and $\Im_h = 2(R_h + E'_+)$ denote the contributions from the transition from the hh band to the conduction band, and $\Re_l = [|M|^2 + |L|^2 - (-R_h + E'_-)^2/3]$ and $\Im_l = 2(-R_h + E'_-)$ for the transition from the lh band to the conduction band. $W_+(\pm\mathbf{k})$ ($W_-(\pm\mathbf{k})$) denotes the transition rate for the left- (right-) circularly polarized 2ω optical field at $\pm\mathbf{k}$, and f_u (f_d) is the occupation number of spin-up (spin-down) electrons in the conduction band at $+\mathbf{k}$ ($-\mathbf{k}$), $P^2 = |\langle S | P_x | X \rangle|^2$, $\eta'_1 = \frac{eA_1}{\omega c m_{cv}}$. Thus, the FR rotation angle of the 2ω pulse per unit length is given by $\theta_F(\omega) = \frac{\omega}{c} \Re(N_+ - N_-)$ [16].

The first two terms in Eq. (5) represent the FR of QUIP arising from the quantum interference between one- and two-photon absorptions. The interference terms are linearly proportional to the wave vector \mathbf{k} (see Eq. (3)), and consequently, FR is proportional to spin current density $\mathbf{J}_i^{\mathbf{s}} = \frac{1}{2} \langle \sigma_{\mathbf{z}} \mathbf{v}_i + \mathbf{v}_i \sigma_{\mathbf{z}} \rangle \sim \sum_{\mathbf{k}} [f_u(\mathbf{k}) + f_d(-\mathbf{k})] \mathbf{k}$, $i, j = x, y, z$. Therefore, the FR of QUIP can detect the spin current directly. The third term in Eq. (5) is the single-photon transition term corresponding to the conventional FR, which is independent of the electron wave vector \mathbf{k} . For the pure spin current, i.e.,

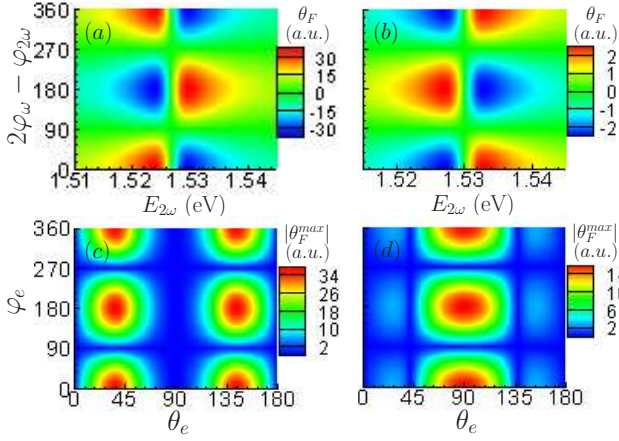


FIG. 2: (color online). Contour plot of FR of QUIP (θ_F) as function of the energy of the 2ω optical field and relative phase of the lights for $\theta_e = 30^\circ$ for the hh -conduction band [(a)] and the lh -conduction band [(b)] transition processes. (c) and (d) show the contour plot of the QUIP FR ($|\theta_F^{max}|$) as function of polar angle θ_e and φ_e of electron wave vector for the hh -conduction band and for the lh -conduction band transition processes at $2\varphi_\omega - \varphi_{2\omega} = 0^\circ$, respectively.

$f_u(\mathbf{k}) = f_d(-\mathbf{k})$, the conventional FR vanishes since the total spin of the pure spin current vanishes. Thus, the conventional FR cannot detect the spin current directly if we neglect the negligible small wave vector of light. The transition rates of one- and two-photon absorptions can be set equal by tuning the intensity of laser beams, i.e., $\eta_1 A_1/2 \simeq A_2$. The FR of QUIP in the detection of pure spin current is on the same order as the conventional FR.

In order to detect the pure spin current $J_j^i(i, j = x, y, z)$ in semiconductor structures, the magnitude and direction of electron velocity and the electron spin orientation are necessary. Figures 2 (a) and (b) show the FR of QUIP as function of the energy and the relative phase between the ω and 2ω optical fields in the presence of a pure spin current in bulk GaAs material[21]. The FR of QUIP reaches its maxima at $2\varphi_\omega - \varphi_{2\omega} = 0^\circ, 180^\circ$, and 360° , and vanishes at $2\varphi_\omega - \varphi_{2\omega} = 90^\circ$ and 270° . One can determine the spin-up and spin-down electron population $f_{u,d}$ at a specific wavevector \mathbf{k} by setting the different phase differences $2\varphi_\omega - \varphi_{2\omega}$ [see Eq. (5)]. The FR of QUIP can also be used to determine the direction of the spin current and the polarization of the laser pulse [see the $\eta_1 \sim \mathbf{k} \cdot \mathbf{a}_1$ term in Eq. (3)], e.g., the FR of QUIP vanishes at $\varphi_e = 90^\circ$ or $\theta_e = 0^\circ$. Figures 2 (c) and (d) describe the maximum of the FR of QUIP as a function of the direction of electron wave vector, i.e., polar angles θ_e and φ_e for $2\varphi_\omega - \varphi_{2\omega} = 0^\circ$. Utilizing the relationship between the FR of QUIP and the direction of the electron wave vector, the direction of the spin current can be determined experimentally. The oscillating FR of QUIP arises from the interplay between the propagation direction of the light and the direction of the electron wave

vector, and the mixing of heavy-hole and light-hole states [see Eq. (5) and $\mathcal{R}_{h,l}$]. The hole mixing effect shortens the oscillation period of the FR of QUIP from 2π to π .

Comparing the left panel with right panel of Figs. 2, the contribution to the FR of QUIP from the $hh_{\pm}-c$ band transition is different from that from the $lh_{\pm}-c$ band transition due to the distinct energy dispersions and the Bloch band-edge wave functions. The distinct energy dispersions make the optical transition occur at different wavevectors for the hh - and lh -c band transitions, while the Bloch band-edge wave functions of the hh and lh bands determine the strengths of the corresponding transitions, i.e., the magnitude of the FR of QUIP.

Now we turn to discuss the detection of pure spin current in the 2D GaAs quantum well[21] in the presence of the spin-orbit interactions (SOIs) [see Fig. 1(c)], i.e., Dresselhaus SOI (DSOI) and Rashba SOI (RSOI). The difference between the 3D and 2D samples is the lifted degeneracy of the hh and lh bands at Γ point and the spin degeneracy caused by the SOIs. We can detect the spin orientation and direction of the pure spin current in the two-dimensional case. Likewise, the FR of QUIP also vanishes when the polarization vector \mathbf{a}_1 is perpendicular to the direction of the pure spin current. Figs. 3(a) and (b) depict the FR of QUIP as a function of the direction of the electron wave vector φ_e and the propagation direction of pulse θ_l for an out-of-plane oriented spin current $J_{\varphi_e}^z$. Figs. 3(c) and (d) show the FR of QUIP as a function of the electron spin orientation and the propagation direction of the laser pulse. When the spin orientation is along the z axis, the FR of QUIP from the hh -c band transition increases since the $|X\rangle$ component of the hh band is crucial for the FR of QUIP. A strongly anisotropic FR can be found in Fig. 3(d) as function of the propagation direction of laser pulses and the spin orientation $\langle \sigma_z \rangle$, which is caused by the anisotropic transition matrix elements $\langle c|\mathbf{p}|v\rangle \sim -\frac{1}{\sqrt{6}}\langle S|\cos\theta_l p_x|X\rangle + \sqrt{\frac{2}{3}}\langle S|\sin\theta_l p_z|Z\rangle$. Thus, we can obtain information about the spin orientation of spin currents by altering the propagation direction of laser pulses. We find that the transition between the lh -c bands leads to a larger FR than that of the hh -c transition since the FR of QUIP for in-plane oriented spin current is dominantly determined by the $|Z\rangle$ component in the hole states. The $|Z\rangle$ component in the band-edge Bloch wave function of the light hole band $|lh\rangle = (1/\sqrt{6})[|X\rangle \pm i|Y\rangle] \otimes |\uparrow\downarrow\rangle - 2|Z\rangle \otimes |\uparrow\downarrow\rangle$ is stronger than that in the heavy hole band $|hh\rangle = (1/\sqrt{2})(|X\rangle \pm i|Y\rangle) \otimes |\uparrow\downarrow\rangle$ in which the $|Z\rangle$ component primarily comes from the hole mixing effect which is strong for large wave vectors. As in 3D sample, the relative phase also can be used to tune the FR of QUIP (see Fig. 3(e) and (f)), which shows a cosine function with the relative phase.

Finally, we can also estimate the magnitude of FR of

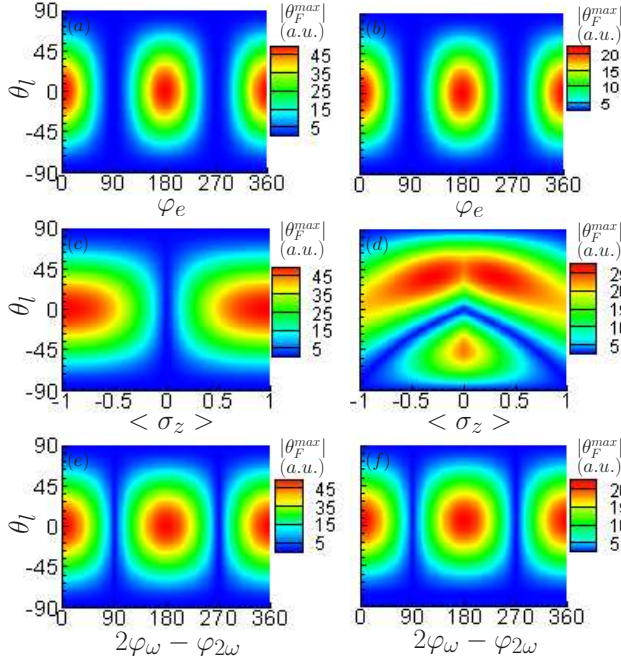


FIG. 3: (color online). Contour plot of FR of QUIP as a function of the direction of electron wave vector and the propagation direction of pulses (a and b), the spin polarization direction and propagation direction of pulses (c and d), the relative phase and propagation direction of pulses (e and f) for the *hh*-c band and *lh*-c band transition processes.

QUIP in realistic semiconductor systems. For the optical quantum interference injection [6] in GaAs QWs, the carrier density is about 10^{17}cm^{-3} , and the corresponding FR of QUIP from our calculations is about $20 \text{mrad}/\mu\text{m}$. In the recent experiment [20], the spin Hall conductivity is about $3\Omega^{-1} \text{m}^{-1}/|e|$ at the electric field $E = 36.2 \text{mV}/\mu\text{m}$, the corresponding spin-up (spin-down) electron density is about 10^{13}cm^{-3} , and the FR of QUIP for this case is about $2\mu\text{rad}/\mu\text{m}$, which is not difficult to detect using the current FR technique. The ultrafast lasers can control quantum system on the femtosecond time scale which is shorter than the electron spin lifetimes and electron momentum relaxation times (about 0.19ps - 1.2ps) [22], especially in the n-type layers, a long spin lifetime ($10^3 - 10^4 \text{ps}$) can be achieved [4, 23].

In summary, we have suggested a new scheme for the direct optical detection of a uniform pure spin current utilizing the two-color momentum-resolved FR of QUIP in bulk semiconductors and two-dimensional semiconductor quantum well structures. By adjusting the incident angle, the polarization direction, the energy, and the relative phase between the ω and 2ω laser pulses, detailed information about the spin current can be detected directly. This scheme may also be important for distinguishing the ESHE and ISHE from the different dependence of the pure spin current on the crystal orientation. Our scheme can also be used to detect the spin polariza-

tion of the charge current.

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